Math 131- Spring 2017- Exam 1

- 14 multiple choice questions worth 5 points each.
- 3 hand graded questions worth 10 points each.

- No notes allowed. You may use one of the approved non-graphing calculators.
- Multiple Choice: Mark your answer on the answer card in pencil.
- Written: To receive full credit, write up a clear, complete solution, showing all steps.

1. Let $f(x) = \sqrt{x^2 - 100}$. Find the domain of f.

(a) $(-\infty, \infty)$
(b) $[10, 10]$
(c) $(10, 10)$
(d) $(-\infty, -10] \cup [10, \infty)$
(e) $(-\infty, -10) \cup (10, \infty)$
(f) $(\infty, 0) \cup (0, \infty)$
(g) none of the above

We require $x^2 - 100 \geq 0$,
so $x \leq -10$ or $x \geq 10$.

2. Let $f(x) = \sqrt{2x+1}$ and $g(x) = \frac{1}{x^2-1}$. Find the domain of $(g \circ f)(x)$.

(a) $(-\infty, \infty)$
(b) $(-\infty, 0) \cup (0, \infty)$
(c) $(-\frac{1}{2}, \infty)$
(d) $[-\frac{1}{2}, \infty)$
(e) $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
(f) $[-\frac{1}{2}, 0) \cup (0, \infty)$
(g) none of the above

$(g \circ f)(x) = g(f(x)) = \dfrac{1}{(\sqrt{2x+1})^2 - 1}$.

Here we must have
① $2x+1 \geq 0$ but also ② $2x+1 \neq 1$
to avoid division by zero.

① $2x+1 \geq 0 \Rightarrow 2x \geq -1 \Rightarrow \boxed{x \geq -\frac{1}{2}}$
and
② $2x+1 \neq 1 \Rightarrow 2x \neq 0 \Rightarrow \boxed{x \neq 0}$

3. By the laws of logarithms,

$$3\log(x+4) - 2\log(y-1) + \frac{1}{2}\log z = \log A$$

where A is which of the following?

(a) $\frac{(x+4)^3}{(y-1)^2\sqrt{z}}$

(b) $\frac{(x+4)^3\sqrt{z}}{(y-1)^2}$

(c) $\frac{(x+4)^3}{(y-1)^2+\sqrt{z}}$

(d) $\frac{(x+4)^3-\sqrt{z}}{(y-1)^2}$

(e) None of the above

$\Downarrow$

$\log(x+4)^3 - \log(y-1)^2 + \log z^{1/2} = \log A$

$\Downarrow$

$\log\left(\frac{(x+4)^3}{(y-1)^2}\cdot\sqrt{z}\right) = \log A$

4. Suppose that for x in the interval $[0,4]$ we know that $\underbrace{4x-4}_{h(x)} \leq f(x) \leq \underbrace{x^2}_{g(x)}$. Find $\lim_{x\to 2} f(x)$.

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

(f) 3

(g) 4

(h) Does not exist

(i) Not enough information is given to determine the value of the limit

Apply the Squeeze Law.

$\lim_{x\to 2} h(x) = 2\cdot 2-4=4$

$\lim_{x\to 2} g(x) = 2^2=4$

& $h(x)\leq f(x)\leq g(x)$,

so $\lim_{x\to 2} f(x) = 4$

5. Let

$$f(x) = \begin{cases} ax+1 & x \le 1 \\ 4-ax^2 & 1 < x. \end{cases}$$

For what value of a will the function be continuous everywhere?

To make f continuous at $X=1$,

require $a \cdot 1 + 1 \left(= \lim_{X\to 1^-} f(x) = f(1)\right) = 4 - a\cdot 1^2 \left(= \lim_{X\to 1^+} f(x)\right)$

(a) $-\frac{1}{2}$
(b) 0
(c) $\frac{1}{2}$
(d) 1
(e) $\frac{3}{2}$
(f) 2
(g) $\frac{5}{2}$
(h) 3
(i) 4
(j) more than one of the above a values will make f continuous
(k) none of the above

$a+1 = 4-a$

$2a = 3$

$a = 3/2$

6. Find the limit:

$$\lim_{x\to 8} \frac{x^2-8x}{x^2-x-56}. = \lim_{X\to 8} \frac{X(X-8)}{(X-8)(X+7)} = \lim_{X\to\infty} \frac{X}{X+7}$$

$$= \frac{8}{8+7}$$

$$= \frac{8}{15}$$

(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{3}{8}$
(e) $\frac{8}{3}$
(f) $\frac{8}{15}$
(g) $\frac{1}{7}$
(h) 8
(i) does not exist
(j) none of the above
(k) Find the limit:

7. Find the limit:

$$\lim_{t\to 0} \frac{\sqrt{100-t^2}-10}{t^2}.$$

(a) $-\frac{1}{20}$
(b) 0
(c) $\frac{1}{20}$
(d) 1
(e) $-\frac{1}{10}$
(f) 2
(g) $\frac{1}{10}$
(h) 3
(i) does not exist
(j) none of the above

$$= \lim_{t\to 0} \frac{(\sqrt{100-t^2}-10)(\sqrt{100-t^2}+10)}{t^2(\sqrt{100-t^2}+10)}$$

$$= \lim_{t\to 0} \frac{100-t^2-100}{t^2(\sqrt{100-t^2}+10)}$$

$$= \lim_{t\to 0} \frac{-\cancel{t^2}}{\cancel{t^2}(\sqrt{100-t^2}+10)} = \frac{-1}{\sqrt{100-0}+10}$$

$$= -\frac{1}{10+10} = -\frac{1}{20}$$

8. Which of the following statements are correct?

I $\frac{x^2+x-6}{x-2} = x+3$ ← Not true for all x (since LHS ONE for X=2)

II $\lim\limits_{x\to 2} \frac{x^2+x-6}{x-2} = \lim\limits_{x\to 2} x+3$ ← Always true, since $X \neq 2$ when $X \to 2$

(a) Both I and II are correct
(b) Both I and II are incorrect
(c) I is correct while II is incorrect
(d) I is incorrect while II is correct
(e) None of the above

9. Find the limit:

$$\lim_{p\to 1} p^4 - 2p^3 + 3p^2 - 4p + 2.$$

(a) -3
(b) -2
(c) -1
(d) 0
(e) 1
(f) 2
(g) 3
(h) 42
(i) None of the above

By direct substitution,

$$\lim_{p\to 1} p^4-2p^3+3p^2-4p+2 = 1-2+3-4+2$$
$$=0$$

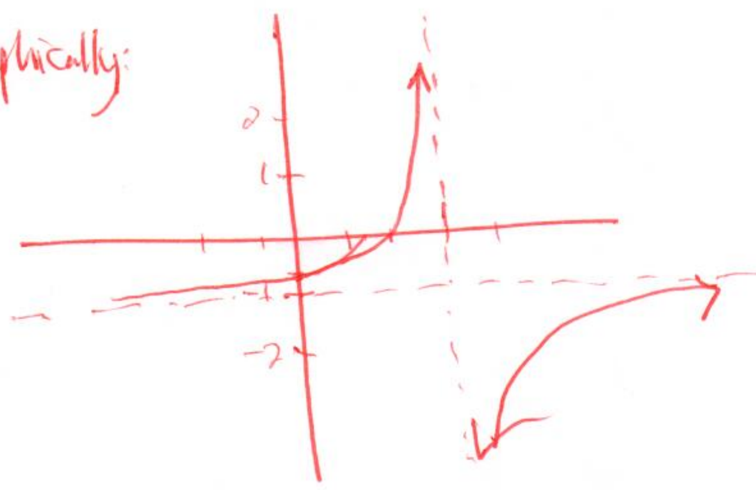
10. Find the limit:

(limit from right)

$$\lim_{y\to 3+} \frac{2-x}{x-3}.$$

(a) $\frac{2}{3}$
(b) $-\frac{2}{3}$
(c) 0
(d) $-\infty$
(e) ∞
(f) None of the above

graphically:



11. Find the limit:

dominant terms

$$\lim_{x\to\infty} \frac{\boxed{3x^5} + 4x^2 + 5}{\boxed{7x^5} + 12x^2 - 2x}.$$

(a) 0

(b) $\frac{3}{7}$

(c) $\frac{7}{3}$

(d) $-\frac{3}{7}$

(e) $-\frac{7}{3}$

(f) $-\infty$

(g) ∞

(h) None of the above

OR

$$= \lim_{x\to\infty} \frac{3 + \frac{4}{x^3} + \frac{5}{x^5}}{7 + \frac{12}{x^3} - \frac{2}{x^4}}$$

$$= \frac{3+0+0}{7+0-0} = \frac{3}{7}$$

12. The distance an object falls (in meters) when dropped from a tall building is given by the function

$$s(t) = 4.9t^2$$

where t is the time in seconds after the release. Find the instantaneous velocity (in m/s) three seconds after the object begins to fall.

(a) 20.0

(b) 24.5

(c) 29.4

(d) 34.3

(e) 49.0

(f) None of the above

$$V(3) = \lim_{h\to 0} \frac{4.9(3+h)^2 - 4.9\,3^2}{h}$$

$$= 4.9 \lim_{h\to 0} \frac{\cancel{3^2} + 6h + h^2 - \cancel{3^2}}{h}$$

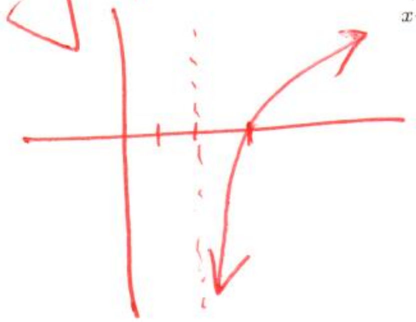
$$= 4.9 \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = 4.9 \cdot 6$$

$$= 29.4$$

13. Find the limit:

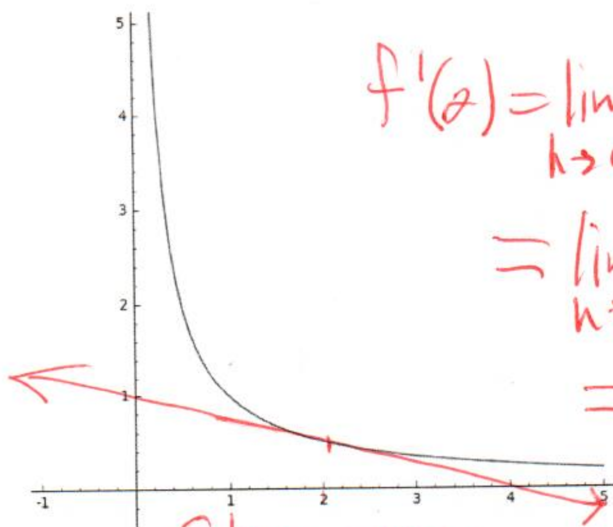
graphically

$$\lim_{x \to 2^+} \ln(x-2).$$



(a) -2
(b) 0
(c) 1
(d) 2
(e) $-\infty$
(f) ∞
(g) None of the above

14. Calculate $f'(a)$ for $f(x) = \frac{1}{x}$ and $a = 2$, then sketch a tangent on the graph of f to check your answer. (The sketch will not be graded, but if it does not match your calculation, find the mistake!)



Slope appears to be $-\frac{1}{4}$

$$f'(2) = \lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$
$$= \lim_{h \to 0} \frac{2-(2+h)}{h(2+h)(2)}$$
$$= \lim_{h \to 0} \frac{-h}{h(2+h)(2)}$$
$$= \frac{-1}{2 \cdot 2} = -\frac{1}{4}$$

(a) -4
(b) -1
(c) $-\frac{1}{4}$
(d) 0
(e) $\frac{1}{4}$
(f) 1
(g) 4
(h) None of the above

Written Problem. Clearly show all steps to receive full credit.

15. Let $f(x) = \dfrac{\sqrt{16x^2+10}}{2x-8}$

(a) Find $\lim_{x\to\infty} f(x)$.

$\sqrt{\frac{1}{x^2}} = \frac{1}{x}$ if $x>0$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\sqrt{16x^2+10}\sqrt{\frac{1}{x^2}}}{(2x-8)\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{16+\frac{10}{x^2}}}{2-\frac{8}{x}}$$

$$= \frac{\sqrt{\lim_{x\to\infty} 16 + \lim_{x\to\infty}\frac{10}{x^2}}}{\lim_{x\to\infty} 2 - \lim_{x\to\infty}\frac{8}{x}} = \frac{\sqrt{16+0}}{2-0} = \frac{4}{2} = \boxed{2}$$

(b) Find $\lim_{x\to-\infty} f(x)$.

$\lim_{x\to-\infty} f(x) = \boxed{-2}$ (Calculation is identical except $\sqrt{\frac{1}{x^2}} = -\frac{1}{x}$ for $x<0$)

(c) List all of the horizontal asymptotes.

$\boxed{y=2,\ y=-2}$

(d) List all of the vertical asymptotes.

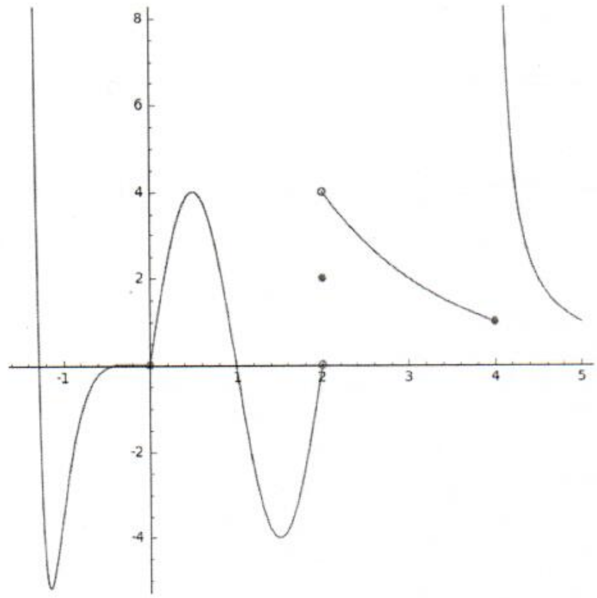
$2x-8=0 \Rightarrow 2x=8$

$x=4$

$\boxed{x=4}$

Written Problem. Clearly show all steps to receive full credit.

16. Use the given graph of $f(x)$ to find the requested limit or function value.



(a) $\lim_{x\to 0^-} f(x) =$ 0

(b) $\lim_{x\to 0^+} f(x) =$ 0

(c) $\lim_{x\to 0} f(x) =$ 0

(d) $\lim_{x\to 2^-} f(x) =$ 0

(e) $\lim_{x\to 2^+} f(x) =$ 4

(f) $\lim_{x\to 2} f(x) =$ DNE

(g) $\lim_{x\to 4^-} f(x) =$ 1

(h) $\lim_{x\to 4^+} f(x) =$ ∞

(i) $f(2) =$ 2

(j) $f(4) =$ 1

Written Problem. Clearly show all steps to receive full credit.

17. For each function below, find $f'(a)$, the derivative of the given function at the number a.

(a) $f(x) = 3x^2 - x + 2$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{3(a+h)^2 - (a+h) + 2 - (3a^2 - a + 2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3a^2} + 6ah + 3h^2 - \cancel{a} - h + \cancel{2} - \cancel{3a^2} + \cancel{a} - \cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6a + 3h - 1)}{\cancel{h}} = 6a + 3\cdot 0 - 1$$

$$= \boxed{6a - 1}$$

(b) $f(x) = \sqrt{4x+1}$

$$f'(a) = \lim_{h\to 0} \frac{\sqrt{4(a+h)+1} - \sqrt{4a+1}}{h} \frac{(\sqrt{4a+4h+1} + \sqrt{4a+1})}{(\sqrt{4a+4h+1} + \sqrt{4a+1})}$$

$$= \lim_{h\to 0} \frac{\cancel{4a} + 4h \cancel{+1} \cancel{-4a} \cancel{-1}}{h(\sqrt{4a+4h+1} + \sqrt{4a+1})}$$

$$= \lim_{h\to 0} \frac{4\cdot \cancel{h}}{\cancel{h}(\sqrt{4a+4h+1} + \sqrt{4a+1})} = \frac{4}{\sqrt{4a+1} + \sqrt{4a+1}}$$

$$= \frac{4}{2\sqrt{4a+1}} = \boxed{\frac{2}{\sqrt{4a+1}}}$$